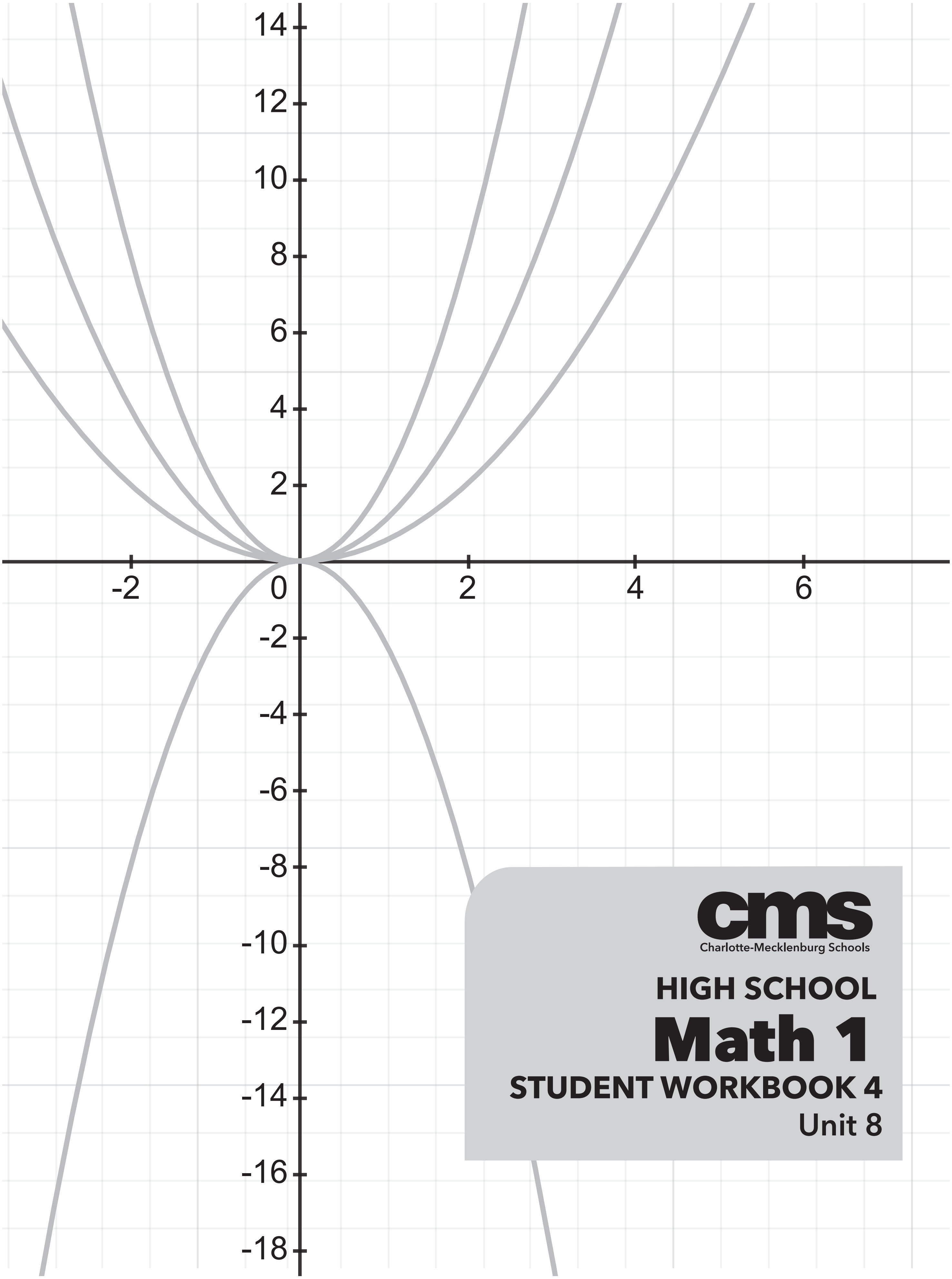

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Charlotte-Mecklenburg Schools

HIGH SCHOOL

Math 1

STUDENT WORKBOOK 4

Unit 8

Table of Contents

Lesson 1: Investigating Sequences

Learning Targets

- I can give an example of a sequence.
- When I see a sequence with a pattern, I can describe a rule for finding the next term of the sequence.

Bridge

Clare Trujillo is graduating from West Mecklenburg High School this year! In her class of 368 graduates, she is number 323 in line. The awarding of diplomas starts at 1:00 p.m. with Andre Abney as the first graduate. Tyler Graham is the 120th graduate to walk across the stage, at 1:12 p.m. Assuming every graduate takes about the same amount of time, at what time should Clare expect to walk across the stage?

Warm-up: What's Next?

Here is a rule for making a list of numbers: Each number is 1 less than twice the previous number.

Pick a number to start with, then follow the rule to build a list of five numbers.

Activity 1: Checker Jumping Puzzle

Some checkers are lined up, with blue on one side (left), red on the other (right), with one empty space between them. A “move” in this checker game pushes any checker forward one space or jumps over any one checker of the other color. Jumping the same color is not allowed, moving backwards is not allowed, and two checkers cannot occupy the same space.

Complete the puzzle by switching the colors completely: ending up with blue on the right, red on the left, with one empty space between them.

1. Using 1 checker on each side, complete the puzzle. What is the smallest number of moves needed?

2. Using 3 checkers on each side, complete the puzzle. What is the smallest number of moves needed?

3. Looking at the table so far, guess the number of moves needed if there are 4 checkers on each side. Then test your guess by solving the puzzle using 4 checkers on each side.

4. How many moves do you think it will take to complete a puzzle with 7 checkers on each side?

Lesson Debrief

Lesson 1 Summary and Glossary

A list of numbers like 3, 5, 7, 9, 11, . . . or 1, 5, 13, 29, 61, . . . is called a **sequence**.

There are many ways to define a sequence, but one way is to describe how each **term** relates to the one before it. For example, the sequence 3, 5, 7, 9, 11, . . . can be described this way: the starting term is 3, then each following term is 2 more than the one before it. The sequence 1, 5, 13, 29, 61, . . . can be described as: the starting term is 1, then each following term is the sum of 3 and twice the previous term.

Throughout this unit, we will study several types of sequences along with ways to represent them.

Sequence: A list of numbers, possibly going on forever, such as all the odd positive integers arranged in order: 1, 3, 5, 7, ...

Term (of a sequence): One of the numbers in a sequence.

Unit 8 Lesson 1 Practice Problems

1. Here is a rule to make a list of numbers: Each number is the sum of the previous two numbers.

   Start with the numbers 0 and 1, then follow the rule to build a sequence of ten numbers.

2. A sequence starts $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$

   a. Give a rule that the sequence could follow.

   b. Follow your rule to write the next three terms in the sequence.

3. A sequence of numbers follows the rule: multiply the previous number by -2 and add 3. The fourth term in the sequence is -7.

   a. Give the next three terms in the sequence.

   b. Give the three terms that came before -7 in the sequence.

4. A sequence starts 0, 5, . . .

   a. Give a rule the sequence could follow and the next three terms for that rule.

   b. Give a **different** rule the sequence could follow and the next three terms for that rule.

5. Consider the expression $(5+x)(6-x)$.

   a. Is the expression equivalent to x^2+x+30? Explain how you know.

   b. Is the expression $30+x-x^2$ in standard form? Explain how you know.

(From Unit 7)

6. Explain or show why the product of a sum of two quantities and a difference of the same two quantities, such as $(2x+1)(2x-1)$, has no linear term when written in standard form.

(From Unit 7)

7. A bank account pays 0.5% monthly interest.

   a. If $500 is put in the account, what will the balance be at the end of one year, assuming no additional deposits or withdrawals are made?

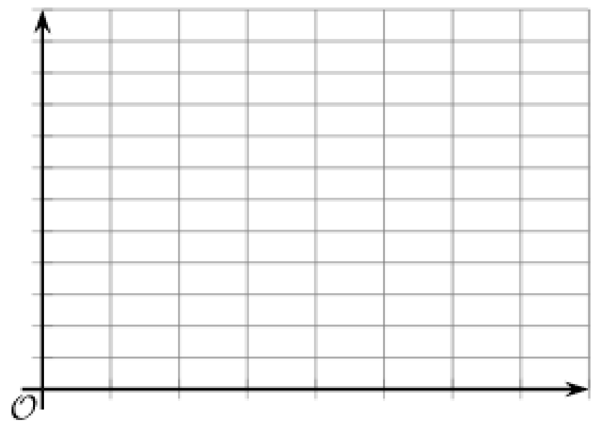
   b. What is the effective annual interest rate?

   c. Is the effective annual interest rate more or less than 6% (the nominal interest rate)?

(From Unit 6)

8. Kiran bought a smoothie every day for a week. Smoothies cost $3 each. The amount of money he spends, in dollars, is a function of the number of days of buying smoothies.

    a. Sketch a graph of this function. Be sure to label the axes.



    b. Describe the domain and range of this function.

(From Unit 5)

9. Select **all** the values for r, the correlation coefficient, that indicate a strong, negative relationship for the line of best fit.

    a. 1
    b. -0.97
    c. -0.45
    d. 0.53
    e. 0.9
    f. -0.8
    g. -1

(From Unit 4)

10. The environmental science club is printing T-shirts for its 15 members. The printing company charges a certain amount for each shirt plus a setup fee of $20.

    If the T-shirt order costs a total of $162.50, how much does the company charge for each shirt?

    (From Unit 3)

Lesson 2: Introducing Geometric Sequences

Learning Targets

- I can create tables and graphs to represent geometric sequences.
- I can find missing terms in a geometric sequence.
- I can determine the common ratio of a geometric sequence.

Warm-up: A Pattern in Lists

What do you notice? What do you wonder?

- 40, 120, 360, 1080, 3240
- 2, 8, 32, 128, 512
- 1000, 500, 250, 125, 62.5
- 256, 192, 144, 108, 81

Activity 1: Pieces of Paper

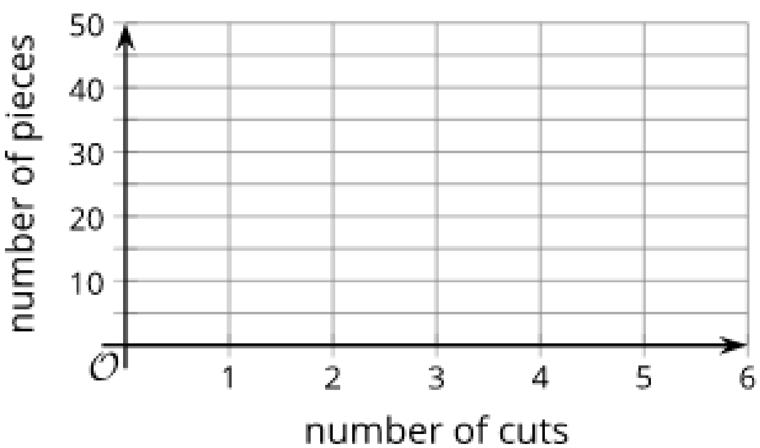
Clare takes a piece of paper, cuts it in half, then stacks the pieces. She takes the stack of two pieces, then cuts in half again to form four pieces, stacking them. She keeps repeating the process.

1. The original piece of paper has length 8 inches and width 10 inches. Complete the table.

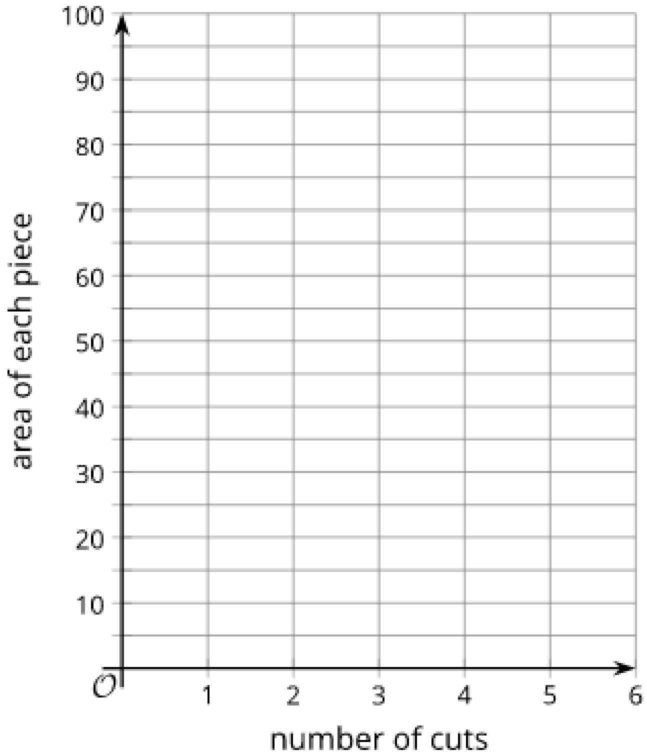
Number of cuts	Number of pieces	Area in square inches of each piece
0		
1		
2		
3		
4		
5		

2. Describe in words how you can use the results after 5 cuts to find the results after 6 cuts.

3. On the given axes, sketch a graph of the number of pieces as a function of the number of cuts. How can you see on the graph how the number of pieces is changing with each cut?



4. On the given axes, sketch a graph of the area of each piece as a function of the number of cuts. How can you see how the area of each piece is changing with each cut?



Are You Ready For More?

1. Clare has a piece of paper that is 8 inches by 10 inches. How many pieces of paper will Clare have if she cuts the paper in half n times? What will the area of each piece be?

2. Why is the product of the number of pieces and the area of each piece always the same? Explain how you know.

Activity 2: Complete the Sequence

Complete each geometric sequence in the first column. Then describe the sequence by filling in the blanks in the second column.

1. 1.5, 3, 6, ______, 24, ______	The starting term is _____. The common ratio is _____ because the current term is ______ times the previous term.
2. 40, 120, 360, ______, ______	The starting term is _____. The common ratio is _____ because the current term is ______ times the previous term.
3. 200, 20, 2, ______, 0.02, ______	The starting term is _____. The common ratio is _____ because the current term is ______ times the previous term.
4. $\frac{1}{7}$, ______, $\frac{9}{7}$, $\frac{27}{7}$, ______	The starting term is _____. The common ratio is _____ because the current term is ______ times the previous term.
5. 24, 12, 6, ______, ______	The starting term is _____. The common ratio is _____ because the current term is ______ times the previous term.

Lesson Debrief

Lesson 2 Summary and Glossary

Consider the sequence 2, 6, 18, . . . How would you describe how to calculate the next term from the previous?

In this case, each term in this sequence is 3 times the term before it.

$$2 \xrightarrow{\cdot 3} 6 \xrightarrow{\cdot 3} 18 \xrightarrow{\cdot 3} \underline{\quad} \xrightarrow{\cdot 3} \underline{\quad}$$

A way to describe this sequence would be: the starting term is 2, and the $current\ term = 3 \cdot previous\ term$.

This is an example of a **geometric sequence**. A geometric sequence is one where the value of each term is the value of the previous term multiplied by a factor. If you know the factor to multiply by, you can use it to find the value of other terms.

> **Geometric sequence**: A sequence in which each term is a constant multiple of the previous term.

This constant multiplier (the “3” in the example) is often called the sequence’s **common ratio**. To find it, you can divide consecutive terms. This can also help you decide whether a sequence is geometric.

> **Common ratio**: The multiplier from one term in a geometric sequence to the next; said another way, the quotient of a term and the previous term

The sequence 1, 3, 5, 7, 9 is not a geometric sequence because $\frac{3}{1} \neq \frac{5}{3} \neq \frac{7}{5}$. The sequence 100, 20, 4, 0.8, however, is geometric because if you divide each term by the previous term you get 0.2 each time: $\frac{20}{100} = \frac{4}{20} = \frac{0.8}{4} = 0.2$.

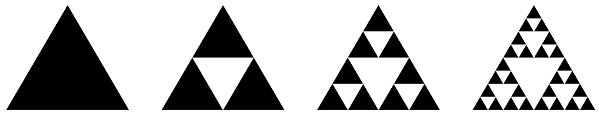
Unit 8 Lesson 2 Practice Problems

1. Here are the first two terms of a geometric sequence: 2, 4. What are the next three terms?

2. What is the common ratio of each geometric sequence?
   a. 1,1,1,1,1
   b. 256, 128, 64
   c. 18, 54, 162
   d. 0.8, 0.08, 0.008
   e. 0.008, 0.08, 0.8

3. Compare and contrast the two geometric sequences listed below by considering the starting term and common ratio of each sequence.
   a. Sequence A: 1, 0.5, 0.25, 0.125, 0.0625
   b. Sequence B: 20, 10, 5, 2.5, 1.25

4. A Sierpinski triangle can be created by starting with an equilateral triangle, breaking the triangle into 4 congruent equilateral triangles, and then removing the middle triangle. Starting from a single black equilateral triangle with an area of 256 square inches, here are the first four steps:



a. Complete this table showing the number of shaded triangles in each step and the area of each triangle.

Step number	**Number of shaded triangles**	**Area of each shaded triangle in square inches**
0	1	256
1	3	
2		
3		
4		
5		

b. Graph the number of shaded triangles as a function of the step number, then separately graph the area of each triangle as a function of the step number.

c. How are these graphs the same? How are they different?

5. The area of a pond covered by algae is $\frac{1}{4}$ of a square meter on Day 1, and it doubles each day. Complete the table.

Day	1	2	3	4	5	6
Area of algae in square meters						

6. Here is a rule to make a list of numbers:

   Each number is 4 less than 3 times the previous number.

   a. Starting with the number 10, build a sequence of five numbers.

   b. Starting with the number 1, build a sequence of five numbers.

   c. Select a different starting number and build a sequence of five numbers.

   (From Unit 8, Lesson 1)

7. A sequence starts 1, -1, . . .

   a. Give a rule the sequence could follow and the next three terms.

   b. Give a **different** rule the sequence could follow and the next three terms.

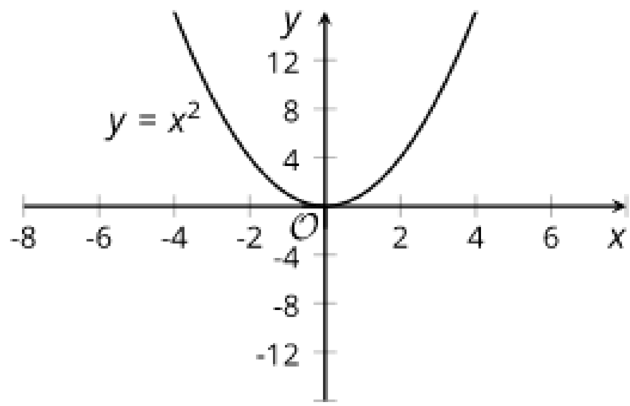
   (From Unit 8, Lesson 1)

8. Which expression in factored form is equivalent to $30x^2 + 31x + 5$?

   a. $(6x + 5)(5x + 1)$
   b. $(5x + 5)(6x + 1)$
   c. $(10x + 5)(3x + 1)$
   d. $(30x + 5)(x + 1)$

(From Unit 7)

9. Here is a graph that represents $y = x^2$.



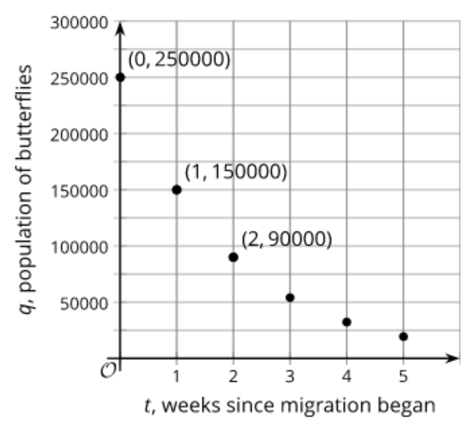
On the same coordinate plane, sketch and label the graph that represents each equation:

   a. $y = -x^2 - 4$

   b. $y = 2x^2 + 4$

(From Unit 7)

10. The graph shows a population of butterflies, t weeks since their migration began.



   a. How many butterflies were in the population when they started the migration? Explain how you know.

   b. How many butterflies were in the population after 1 week? What about after 2 weeks?

   c. Write an equation for the population, q, after t weeks.

(From Unit 6)

11. A person owes $1000 on a credit card that charges an interest rate of 2% per month.

Complete this table showing the credit card balance each month if they do not make any payments.

Month	Total bill in dollars
1	1,000
2	1,020
3	1,040.40
4	
5	
6	
7	
8	

(From Unit 6)

12. Which equation has exactly one solution in common with the equation $y = 6x - 2$?

a. $18x - 3y = 6$

b. $\frac{1}{2}y = 3x - 2$

c. $2y = 4x - 12$

d. $18x - 12 = 3y$

(From Unit 3)

Lesson 3: Different Types of Sequences

Learning Targets

- I can compare and contrast arithmetic and geometric sequences.
- I can calculate the rate of change of an arithmetic sequence.
- I can tell whether a sequence is arithmetic or geometric by looking at a table or graph.

Bridge

One of the tables below represents a linear function, and one does not. Which one represents a linear function? How do you know?

1.

x	0	10	20	30	40	50
y	2.8	10.2	17.6	25	32.4	39.8

2.

x	0	10	20	30	40	50
y	5.6	10.8	16.2	21.8	27.6	33.6

Warm-up: Revisiting Function Notation

Consider the function f given by $f(n) = 3n - 7$. This function takes an input, multiplies it by 3, then subtracts 7.

Evaluate mentally.

1. $f(10)$
2. $f(10) - 1$
3. $f(10 - 1)$
4. $f(5) - f(4)$

Activity 1: Three Sequences

Here are the values of the first five terms of three sequences:

- A: 30, 40, 50, 60, 70, . . .
- B: 0, 5, 15, 30, 50, . . .
- C: 1, 2, 4, 8, 16, . . .

1. For each sequence, describe a way to produce a new term from the previous term.

2. If the patterns you described continue, which sequence has the smallest value for the 10^{th} term?

3. Which of these could be geometric sequences? Explain how you know.

Are You Ready For More?

Elena says that it's not possible to have a sequence of numbers that is *both* arithmetic and geometric. Do you agree with Elena? Explain your reasoning.

Activity 2: Representing a Sequence

Jada and Mai are trying to decide what type of sequence this could be:

Term number	Value
1	2
2	6
5	18

Jada says: “I think this sequence is geometric because in the value column each row is 3 times the previous row.”

Mai says: “I don’t think it is geometric. I graphed it, and it doesn’t look geometric.”

Do you agree with Jada or Mai? Explain or show your reasoning.

Lesson Debrief

Lesson 3 Summary and Glossary

Consider the sequence 2, 5, 8, . . . How would you describe how to calculate the next term from the previous?

In this case, each term in this sequence is 3 more than the term before it.

$$2 \xrightarrow{+3} 5 \xrightarrow{+3} 8 \xrightarrow{+3} \_\_ \xrightarrow{+3} \_\_$$

A way to describe this sequence is: the starting term is 2 and the $current\ term = previous\ term + 3$.

This is an example of an **arithmetic sequence**. An arithmetic sequence is one where each term is the sum of a previous term and a constant. (In the previous example, the constant is 3.) If you know the constant to add, you can use it to find other terms.

> **Arithmetic sequence:** A sequence in which each term comes from adding a constant to the previous term.

For example, each term in this sequence is 3 more than the term before it. To find this constant, called the **common difference**, you can subtract consecutive terms. This can also help you decide whether a sequence is arithmetic.

> **Common difference**: The number to add to get from one term in an arithmetic sequence to the next; said another way, the difference of a term and the previous term.

For example, the sequence 3, 6, 12, 24 is not an arithmetic sequence because $6-3 \neq 12-6 \neq 24-12$. But the sequence 100, 80, 60, 40 is because if the differences of consecutive terms are all the same: $80-100=60-80=40-60=-20$. This means that the rate of change is -20 for the sequence 100, 80, 60, 40.

It is important to remember that while the last two lessons have introduced geometric and arithmetic sequences, there are many other sequences that are neither geometric nor arithmetic.

Unit 8 Lesson 3 Practice Problems

1. The first two terms of some different arithmetic sequences are listed below. What are the next three terms of each sequence?

    a. -2, 4

    b. 11, 111

    c. 5, 7.5

    d. 5, -4

2. For each sequence, decide whether it could be arithmetic, geometric, or neither.

    a. 200, 40, 8, . . .

    b. 2, 4, 16, . . .

    c. 10, 20, 30, . . .

    d. 100, 20, 4, . . .

    e. 6, 12, 18, . . .

3. Complete each arithmetic sequence with its missing terms, then state the common difference for each sequence.

   a. -3, -2, _____, _____, 1

   b. _____, 13, 25, _____, _____

   c. 1, .25, _____, -1.25, _____

   d. 92, _____, _____ ,_____, 80

4. A sequence starts with the terms 1 and 10.

   a. Find the next two terms if it is arithmetic: 1, 10, _____, _____.

   b. Find the next two terms if it is geometric: 1, 10, _____, _____.

   c. Find two possible next terms if it is neither arithmetic nor geometric: 1, 10, _____, _____.

5. For each sequence, decide whether it could be arithmetic, geometric, or neither.

   a. 25, 5, 1, …

   b. 25, 19, 13, …

   c. 4, 9, 16, …

   d. 50, 60, 70, …

   e. $\frac{1}{2}$, 3, 18, …

6. Complete each geometric sequence with the missing terms. Then find the common ratio for each.

   a. _____, 5, 25, _____, 625

   b. -1, _____, -36, 216, _____

   c. 10, 5, _____, _____, 0.625

   d. _____, _____, 36, -108, _____

   e. _____, 12, 18, 27, _____

(From Unit 8, Lesson 2)

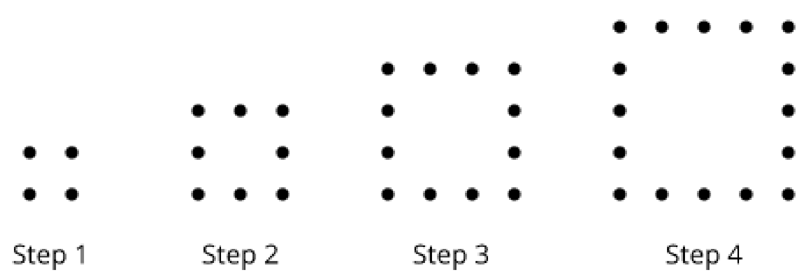
7. The first term of a sequence is 4.

   a. Choose a common ratio and list the next three terms of a geometric sequence.

   b. Choose a **different** common ratio and list the next three terms of a geometric sequence.

(From Unit 8, Lesson 2)

8. Han says this pattern of dots can be represented by a quadratic relationship because the dots are arranged in a rectangle in each step.

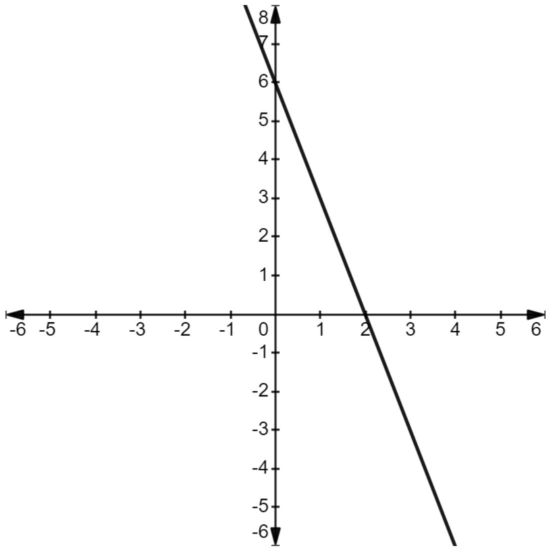


Do you agree? Explain your reasoning.

(From Unit 7)

9. Review the functions below.

Function F



Function G

$g(x) = \frac{1}{2}(x+2)(x-4)$

a. Over what domain are both functions decreasing?

b. Which function has the larger y-intercept?

(From Unit 7)

10. Match each quadratic expression given in factored form with an equivalent expression in standard form. One expression in standard form has no match.

a. $(2+x)(2-x)$

b. $(x+9)(x-9)$

c. $(2+x)(x-2)$

d. $(x+y)(x-y)$

1. $x^2 - 4$
2. $81 - x^2$
3. $x^2 - y^2$
4. $4 - x^2$
5. $x^2 - 81$

(From Unit 7)

11. Solve each system of equations.

a. $\begin{cases} 7x - 12y = 180 \\ 7x = 84 \end{cases}$

b. $\begin{cases} -16y = 4x \\ 4x + 27y = 11 \end{cases}$

(From Unit 3)

12. Does the following table represent a linear or non-linear function? Explain your answer.

x	0	1	3	6	8
y	5	10	15	20	25

(Addressing NC.8.F.3)

Lesson 4: Sequences Are Functions

Learning Targets

- I can describe a sequence as a function and give its domain.
- I can define arithmetic and geometric sequences recursively using function notation.

Bridge

What are the domains of the functions below? Explain your answers.

1. Students' scores on Ms. Starks' Math 1 test are modeled by a function that outputs a score based on the number of questions answered correctly. There are 12 questions on the test.

2. A medical research facility conducted a study using a function to determine healthy body weight based on patients' heights. The patients' heights in the study varied between 58 inches and 79 inches.

Warm-up: Bowling for Triangles (Part One)

Use the sentence frame to describe how to produce one figure of the pattern from the previous figure by completing the provided sentence.

Figure 1 **Figure 2** **Figure 3** **Figure 4**

The starting term is ______, and the current term = ____ .

Activity 1: Bowling for Triangles (Part Two)

Here is a visual pattern of dots. The number of dots $D(n)$ is a function of the figure number n.

Figure 1

Figure 2

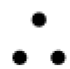


Figure 3

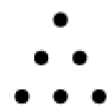
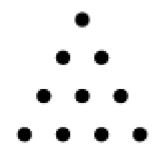


Figure 4



1. What values make sense for n in this situation? Would $n = 2.5$ make sense?

2. Complete the table for Figures 1 to 5.

n	$D(n)$
1	1
2	$D(1) + 2 = 3$
3	$D(2) + 3 = 6$
4	
5	

3. Following the pattern in the table, write an equation for $D(n)$ in terms of the previous figure. Be prepared to explain your reasoning.

Are You Ready For More?

Consider the same triangular pattern.

1. Is the sequence defined by the number of dots in each step arithmetic, geometric, or neither? Explain how you know.

2. Can you write an expression for the number of dots in Step n without using the value of D from a previous step?

Activity 2: Let's Define Some Sequences

Use the first five terms of each sequence to state if the sequence is arithmetic, geometric, or neither. Next, define the sequence recursively using function notation.

1. A: 30, 40, 50, 60, 70, . . .
2. B: 1, 2, 4, 8, 16, 32, . . .
3. C: $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$
4. D: 20, 13, 6, -1, -8, . . .
5. E: 1, 3, 7, 15, 31, . . .

Lesson Debrief

Lesson 4 Summary and Glossary

Sometimes we can define a sequence recursively. That is, we can describe how to calculate the next term in a sequence if we know the previous term.

Here's a sequence: 6, 10, 14, 18, 22, . . . This is an arithmetic sequence, where each term is 4 more than the previous term. Since sequences are functions, let's call this sequence f .Then we can use function notation to write $f(n) = f(n-1) + 4$. Here, $f(n)$ is the term, $f(n-1)$ is the previous term, and + 4 represents the common difference.

When we define a function recursively, we also must say what the first term is. Without that, there would be no way of knowing if the sequence defined by $f(n) = f(n-1) + 4$ started with 6 or 81 or any other number. Here, one possible starting value is $f(1) = 6$. (It could also make sense to number the terms starting with 0, using $f(0) = 6$, and we'll talk more about this later.)

Combining this information gives the recursive definition: $f(1) = 6$ and $f(n) = f(n-1) + 4$ for $n \geq 2$, where n is an integer. We include the $n \geq 2$ at the end since the value of f at 1 is already given and the other terms in the sequence are generated by inputting integers larger than 1 into the definition.

Recursive definition (of a sequence): A way of writing a rule for the terms of sequence that depends on the terms that came before. For example, the sequence a: 10, 8, 6, 4… has recursive definition $a(1) = 10$ and $a(n) = a(n-1) - 2$ for $n \geq 2$.

Unit 8 Lesson 4 Practice Problems

1. Match each sequence with one of the definitions. Note that only the part of the definition showing the relationship between the current term and the previous term is given so as not to give away the solutions.

   a. 6, 12, 18, 24

   b. 2, 14, 98, 686

   c. 160, 80, 40, 20

   1. $a(n) = 7 \cdot a(n-1)$

   2. $b(n) = \frac{1}{2} \cdot b(n-1)$

   3. $c(n) = c(n-1) + 6$

2. Write the first five terms of each sequence. Determine whether each sequence is arithmetic, geometric, or neither.

   a. $a(1) = 7, a(n) = a(n-1) - 3$ for $n \geq 2$.

   b. $b(1) = 2, b(n) = 2 \cdot b(n-1) - 1$ for $n \geq 2$.

   c. $c(1) = 3, c(n) = 10 \cdot c(n-1)$ for $n \geq 2$.

   d. $d(1) = 1, d(n) = n \cdot d(n-1)$ for $n \geq 2$.

3. Match each recursive definition with one of the sequences.

a. $h(1) = 1, h(n) = 2 \cdot h(n-1) + 1$ for $n \geq 2$

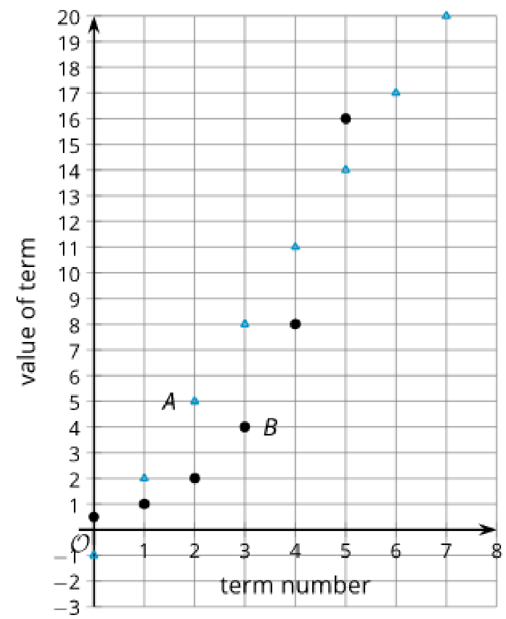
b. $p(1) = 1, p(n) = 2 \cdot p(n-1)$ for $n \geq 2$

c. $a(1) = 80, a(n) = \frac{1}{2} \cdot a(n-1)$ for $n \geq 2$

1. 80, 40, 20, 10, 5
2. 1, 2, 4, 8, 16
3. 1, 3, 7, 15, 31

4. Here is the graph of two sequences. Complete the table for each sequence.

Term number	Sequence A	Sequence B
0	-1	$\frac{1}{2}$
1		
2		
3		
4		
5		
6		



a. For sequence A, describe a way to produce a new term from the previous term.

b. For sequence B, describe a way to produce a new term from the previous term.

c. Which of these is a geometric sequence? Explain how you know.

(From Unit 8, Lesson 3)

5. The first five terms of some sequences are given. State a rule that each sequence could follow.

   a. 2, 4, 6, 8, 10

   b. 5, 7, 9, 11, 13

   c. 50, 25, 0, -25, -50

   d. $\frac{1}{3}$, 1, 3, 9, 27

(From Unit 8, Lesson 1)

6.

   a. Describe the graph of $y = -x^2$. (Does it open upward or downward? Where is its y-intercept? What about its x-intercepts?)

   b. Without graphing, describe how adding $16x$ to $-x^2$ would change each feature of the graph of $y = -x^2$. (If you get stuck, consider writing the expression in factored form.)

      i. the x-intercepts

      ii. the vertex

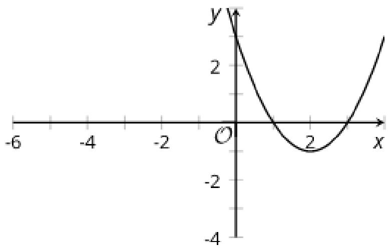
      iii. the y-intercept

      iv. the direction of opening of the U-shape graph

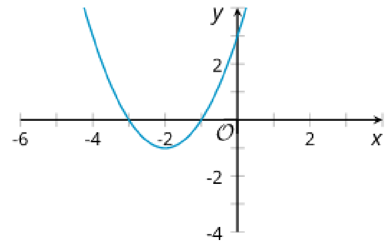
(From Unit 7)

7. Here are four graphs. Match each graph with a quadratic equation that it represents.

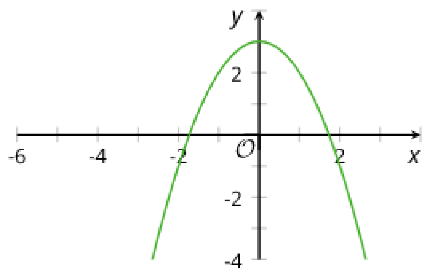
Graph A



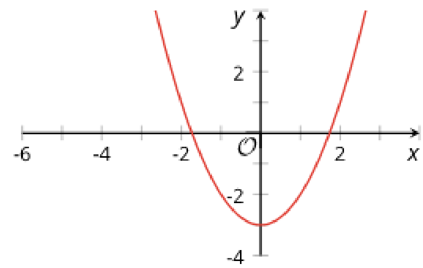
Graph B



Graph C



Graph D



a. $y = -x^2 + 3$

b. $y = (x + 1)(x + 3)$

c. $y = x^2 - 3$

d. $y = (x - 1)(x - 3)$

(From Unit 7)

8. What are the x-intercepts of the graph of the function defined by $(x - 2)(2x + 1)$?

a. $(2, 0)$, and $(-1, 0)$

b. $(2, 0)$ and $(-\frac{1}{2}, 0)$

c. $(-2, 0)$ and $(1, 0)$

d. $(-2, 0)$ and $(\frac{1}{2}, 0)$

(From Unit 7)

9. (*Technology required*). Function h is defined by $h(x) = 5x + 7$, and function k is defined by $k(x) = (1.005)^x$.

    a. Complete the table with values of $h(x)$ and $k(x)$. When necessary, round to 2 decimal places.

x	$h(x)$	$k(x)$
1		
10		
50		
100		

    b. Which function do you think *eventually* grows faster? Explain your reasoning.

    c. Use graphing technology to verify your answer to the previous question.

(From Unit 6)

10. At 6:00 a.m., Lin began hiking. At noon, she had hiked 12 miles. At 4:00 p.m., Lin finished hiking with a total trip of 26 miles.

    During which time interval was Lin hiking faster? Explain how you know.

(From Unit 5)

11. Function f is defined by $f(x) = 2x - 7$, and g is defined by $g(x) = 5^x$.

   a. Find $f(3)$, $f(2)$, $f(1)$, $f(0)$, and $f(-1)$.

   b. Find $g(3)$, $g(2)$, $g(1)$, $g(0)$, and $g(-1)$.

(From Unit 5)

Lesson 5: Representing Sequences

Learning Targets

- I can interpret sequences that are expressed in function or subscript notation.
- I can define arithmetic and geometric sequences using subscript notation.

Bridge

1. Evaluate each expression given $n = 4$.
   a. $n - 1$

   b. n

   c. $n + 1$

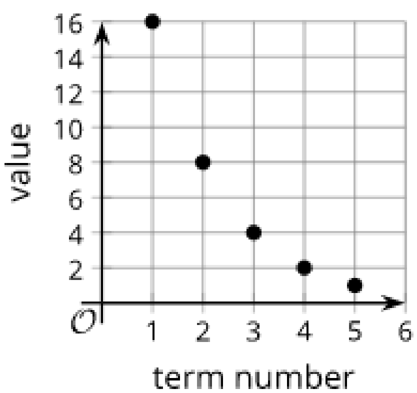
2. What do you notice about the values of the three expressions? Explain why this is true.

Warm-up: Reading Representations

For each sequence shown, find either the common ratio or common difference. Be prepared to explain your reasoning.

1. 5, 15, 25, 35, 45, . . .

2. Starting at 10, each new term is $\frac{5}{2}$ less than the previous term.

3. 

4. $g(1) = -5, \quad g(n) = g(n-1) \cdot -2$ for $n \geq 2$

5.

n	$f(n)$
1	0
2	0.1
3	0.2
4	0.3
5	0.4

Activity 1: Matching Recursive Definitions

Take turns with your partner to match a sequence with a recursive definition. It may help to first figure out if the sequence is arithmetic or geometric.

- For each match that you find, explain to your partner how you know it's a match.
- For each match that your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.

There is one sequence and one definition that do not have matches. Create their corresponding match.

Sequences:

1. 3, 6, 12, 24
2. 18, 36, 72, 144
3. 3, 8, 13, 18
4. 18, 13, 8, 3
5. 18, 9, 4.5, 2.25
6. 18, 20, 22, 24

Definitions:

- $G(1) = 18, \;\; G(n) = \frac{1}{2} \cdot G(n-1), \;\; n \geq 2$
- $H(1) = 3, \;\; H(n) = 5 \cdot H(n-1), \;\; n \geq 2$
- $J(1) = 3, \;\; J(n) = J(n-1) + 5, \;\; n \geq 2$
- $K(1) = 18, \;\; K(n) = K(n-1) - 5, \;\; n \geq 2$
- $L(1) = 18, \;\; L(n) = 2 \cdot L(n-1), \;\; n \geq 2$
- $M(1) = 3, \;\; M(n) = 2 \cdot M(n-1), \;\; n \geq 2$

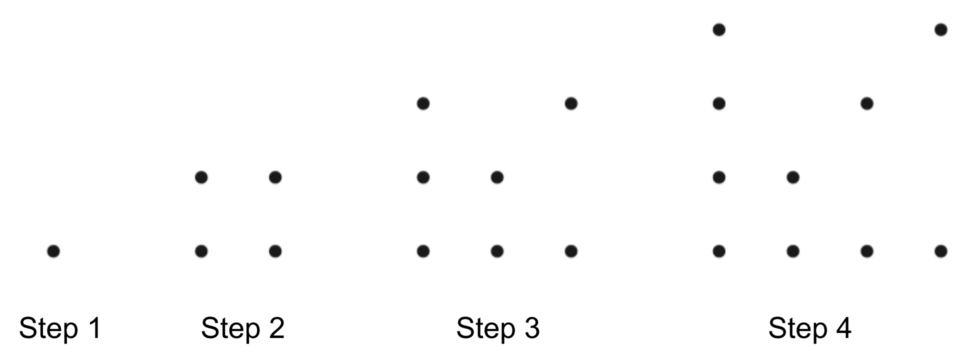
Activity 2: Subscript Notation

1. For each of the following recursive definitions, calculate the first four terms of the sequence it represents and identify if it is geometric or arithmetic.

   a. $a_1 = 4, a_n = a_{n-1} \cdot 5;\ n \geq 2$

   b. $a = 30, a_n = a_{n-1} - 2;\ n \geq 2$

2. Here is a pattern where the number of dots increases with each new step. Write a recursive definition for the total number of dots d_n in Step n.



Lesson Debrief

Lesson 5 Summary and Glossary

We can define sequences recursively using function notation or subscript notation. For the sequence d with terms 4, 7, 10, 13, 16, ..., the starting term is 4, and the constant rate of change is 3.

Function Notation	Subscript Notation
$d(1) = 4, d(n) = d(n-1) + 3$ for $n \geq 2$. $d(1)$ represents the first term of the sequence. $d(n)$ represents the current term in the sequence $d(n-1)$ represents the previous term in the sequence	$d_1 = 4, d_n = d_{n-1} + 3$ for $n \geq 2$. d_1 represents the first term of the sequence d_n represents the current term in the sequence d_{n-1} represents the previous term in the sequence

This type of definition tells us how to find any term, n, if we know the previous term, $n-1$. It is not as helpful in calculating terms that are far away like $d(100)$ or d_{100}. Some sequences do not have recursive definitions, but geometric and arithmetic sequences always do.

Unit 8 Lesson 5 Practice Problems

1. An arithmetic sequence a starts 2, 5, . . .

   a. Write a recursive definition for this sequence using function notation.

   b. Use your definition to find $a(6)$.

2. A geometric sequence g starts 1, 3, . . .

   a. Write a recursive definition for this sequence using subscript notation.

   b. Explain how to use the recursive definition to determine g_{30}. (Don't actually determine the value.)

3. Match each sequence with one of the recursive definitions. Note that only the part of the definition showing the relationship between the current term and the previous term is given so as not to give away the solutions.

   a. 3, 15, 75, 375

   b. 18, 6, 2, $\frac{2}{3}$

   c. 1, 2, 4, 7

   d. 17, 13, 9, 5

   1. $a(n) = \frac{1}{3} \cdot a(n-1)$
   2. $b(n) = b(n-1) - 4$
   3. $c_n = 5 \cdot c_{n-1}$
   4. $d_n = d_{n-1} + (n-1)$

4. Match each sequence with one of the recursive definitions.

a. ...3, -9, -21, -33

b. ...3, 5, 7, 9

c. ...3, 10, 25, 56

c. ...3, -9, 27, -81

1. $a_n = a_{n-1} + 2,\ a_1 = -3$
2. $a_n = a_{n-1} \cdot (-3),\ a_1 = -1$
3. $a_n = a_{n-1} - 12,\ a_1 = 27$
4. $a_n = 2a_{n-1} + n,\ a_1 - 1$

5. A geometric sequence g starts 80, 40, …

   a. Write a recursive definition for this sequence using function notation.

   b. Use your definition to make a table of values for $g(n)$ for the first six terms.

   c. Explain how to use the recursive definition to find $g(100)$. (Don't actually determine the value.)

6. Here is a table showing values of sequence p. Define p recursively using function notation.

n	$p(n)$
1	5,000
2	500
3	50
4	5
5	0.5

7. Write the first five terms of each sequence.

   a. $a(1) = 1$, $a(n) = 3 \cdot a(n-1)$, $n \geq 2$

   b. $b(1) = 1$, $b(n) = -2 + b(n-1)$, $n \geq 2$

   c. $c(1) = 1$, $c(n) = 2 \cdot c(n-1) + 1$, $n \geq 2$

   d. $d(1) = 1$, $d(n) = d(n-1)^2 + 1$, $n \geq 2$

   e. $f(1) = 1$, $f(n) = f(n-1) + 2n - 2$, $n \geq 2$

(From Unit 8, Lesson 4)

8. A sequence has $f(1) = 120, f(2) = 60$.

   a. Determine the next two terms if it is an arithmetic sequence, then write a recursive definition that matches the sequence in the form $f(1) = 120$, $f(n) = f(n-1) + \_$ for $n \geq 2$.

   b. Determine the next two terms if it is a geometric sequence, then write a recursive definition that matches the sequence in the form $f(1) = 120$, $f(n) = \_ \cdot f(n-1)$ for $n \geq 2$.

(From Unit 8, Lesson 4)

9. One hour after an antibiotic is administered, a bacteria population is 1,000,000. Each subsequent hour, it decreases by a factor of $\frac{1}{2}$.

   a. Complete the table with the bacteria population at the given times.

Number of hours	**Population**
1	1,000,000
2	
3	
4	
5	
6	

   b. Do the bacteria populations make a geometric sequence? Explain how you know.

(From Unit 8, Lesson 2)

10. Rewrite each expression in standard form.

a. $(x+3)(x-3)$

b. $(7+x)(x-7)$

c. $(2x-5)(2x+5)$

d. $(x+\frac{1}{8})(x-\frac{1}{8})$

(From Unit 7)

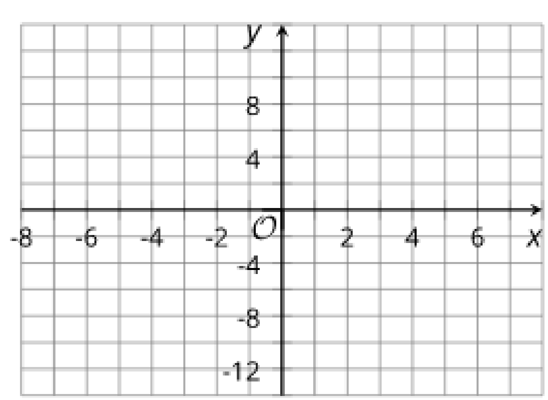
11. Consider the equation $y=2x(6-x)$.

a. What are the x-intercepts of the graph of this equation? Explain how you know.

b. What is the x-coordinate of the vertex of the graph of this equation? Explain how you know.

c. What is the y-coordinate of the vertex? Show your reasoning.

d. Sketch the graph of this equation.



(From Unit 7)

12. (*Technology required*). A moth population, p, is modeled by the equation $p = 500,000 \cdot (\frac{1}{2})^w$, where w is the number of weeks since the population was first measured.

    a. What was the moth population when it was first measured?

    b. What was the moth population after 1 week? What about 1.5 weeks?

    c. Use technology to graph the population over time and find out when it falls below 10,000.

(From Unit 6)

Lesson 6: The n^{th} Term

Learning Targets

- I can interpret an equation for the n^{th} term of a sequence.
- I can explain why different equations can represent the same sequence.

Bridge

1. Write a linear equation that has a constant rate of change of 6 and passes through the point $(1, 3)$.

2. Write a linear equation that has a constant rate of change of -2 and passes through the point $(0, 10)$.

Warm-up: Repeated Operations

Which one doesn't belong? Explain your reasoning.

a. $5+2+2+2+2+2+2$	b. $5+2(6)$
c. $5\cdot 2^6$	d. $5\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2$

Activity 1: More Pieces of Paper

1. Clare takes a piece of paper with length 8 inches and width 10 inches and cuts it in half. Then she cuts it in half again, and again...

    a. Instead of writing a recursive definition, Clare writes $C(n) = 80 \cdot (\frac{1}{2})^n$, where $C(n)$ is the area, in square inches, of the paper after n cuts. Explain where the different terms in her expression came from.

    b. Approximately what is the area of the paper after 10 cuts? What is the area after 25 cuts?

2. Kiran has a piece of paper with length 8 inches and width 10 inches. He cuts off one end of the paper, making a strip that is 1 inch by 8 inches. Then he does it again, and again...

   a. Complete the table for the area of Kiran's paper, $K(n)$, in square inches, after n cuts.

n	$K(n)$
0	80
1	
2	$80-8-8=80-8(2)=64$
3	
4	
5	

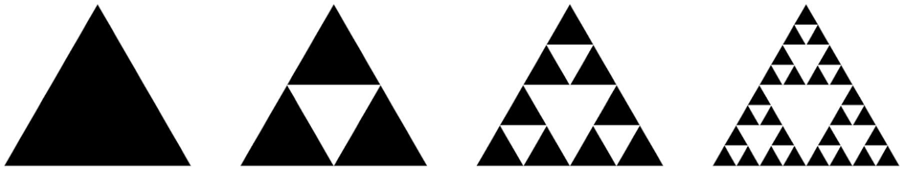
   b. Kiran says the area after 6 cuts, in square inches, is $80-8\cdot 6$. Explain where the different terms in his expression came from.

   c. Write a definition for $K(n)$ that is not recursive.

3. Which is greater, $K(6)$ for Kiran's function or $C(6)$ for Clare's function?

Activity 2: A Sierpinski Triangle

A Sierpinski triangle can be created by starting with an equilateral triangle, breaking the triangle into four congruent equilateral triangles, and then removing the middle triangle. Starting from a single black equilateral triangle:



1. Let S be the number of black triangles in Step n. Define $S(n)$ recursively.

2. Andre and Lin are asked to write an equation for S that isn't recursive. Andre writes $S(n) = 3^n$ for $n \geq 0$, while Lin writes $S(n) = 3^{n-1}$ for $n \geq 1$. Whose equation do you think is correct? Explain or show your reasoning.

Are You Ready For More?

Here is a geometric sequence. Find the missing terms.

3, _____, 6, _____, 12, _____, 24

Activity 3: Recursive and Explicit Definitions

1. Complete the following table. In each case, let the initial value be represented by a_1 or $a(1)$.

Sequence	Recursive Definition	Explicit Definition
a. $9, 15, 21, 27, \ldots$		
b.	$a_1 = 12$, $a_n = a_{n-1} \cdot 3$, $n \geq 2$	
c.		$a(n) = 120(\frac{1}{2})^n$ for $n \geq 1$
d.		$a(n) = 14 - 9n$ for $n \geq 1$

2. Noah and Han are writing an explicit definition for the sequence defined by $a_1 = 16$, $a_n = a_{n-1} + 7$, $n \geq 2$. Noah wrote $a(n) = 16 + 7n$, and Han wrote $a(n) = 9 + 7n$.

   a. Decide which explicit definition is correct and explain how you know.

   b. For the explicit definition that is incorrect, explain the mistake and how to correct it.

Are You Ready For More?

Another way to write an explicit definition for Noah and Han's sequence is $a(n) = 16 + 7(n-1)$, $n \geq 1$

1. Use this definition to calculate the first, second, and third term of the sequence. Why do we need to write "$n-1$" instead of "n"?

2. Use point-slope form to write an equation for a line that passes through the point $(1, 16)$ and has slope 7. What do you notice?

Lesson Debrief

Lesson 6 Summary and Glossary

Here is an arithmetic sequence f: 6, 10, 14, 18, 22, ...

In this sequence, each term is 4 more than the previous term. One recursive definition of this sequence is $f_1 = 6$, $f_1 = f_{n-1} + 4$ for $n \geq 2$. We could also write $f_0 = 6$, $f_1 = f_{n-1} + 4$ for $n \geq 1$ since it generates the same sequence. Neither of these definitions is better than the other; we just have to remember how we chose to define the “first term” of the sequence: f_1 or f_0. Let's use f_1 for now.

While defining a sequence recursively works to calculate the current term from the previous, if we wanted to calculate, say, f_{100}, it would mean calculating all the terms up to f_{99} to get there! Let's think of a better way.

Since we know that each term has an increasing number of fours, we could write the terms of f organized in a table like the one shown here.

n	$f(n)$
1	$6+0=6+4(0)=6$
2	$6+4=6+4(1)=10$
3	$6+4+4=6+4(2)=14$
4	$6+4+4+4=6+4(3)=18$
5	$6+4+4+4+4=6+4(4)=22$

Looking carefully at the pattern in the table, we can say that for the n^{th} term $f(n) = 6 + 4(n-1)$ for $n \geq 1$. This is sometimes called an **explicit** or **closed-form definition** of a sequence, but it's really just a way to calculate the value of the n^{th} term without having to calculate all the terms that came before it. Need to know $f(100)$? Just compute $6 + 4(100-1)$. Defining an arithmetic sequence this way takes advantage of the fact that this type of sequence is a linear function with a starting value (in this case, 6) and rate of change (in this case, 4). If we had decided to start the sequence at $n = 0$ so that $f(0) = 6$, we would have written the equation for the n^{th} term as $f(n) = 6 + 4(n)$ for $n \geq 0$.

Geometric sequences behave the same way, but with repeated multiplication. The geometric sequence g: 3, 15, 75, 375, . . . can be written as 3, $3 \cdot 5$, $3 \cdot 5 \cdot 5$, $3 \cdot 5 \cdot 5 \cdot 5$, ... This means if $g(0) = 3$, we can define the n^{th} term directly as $g(n) = 3 \cdot 5^n$.

Explicit (or closed-form) definition of a sequence: A way of writing a rule for the terms of sequence that does not depend on the terms that came before. For example, the sequence a: 10, 8, 6, 4… has explicit definition $a(n) = 10 - 2(n-1)$, where $n \geq 1$.

Unit 8 Lesson 6 Practice Problems

1. A sequence is defined by $f(0) = -20$, $f(n) = f(n-1) - 5$ for $n \geq 1$.

   a. Explain why $f(1) = -20 - 5$.

   b. Explain why $f(3) = -20 - 5 - 5 - 5$.

   c. Complete the expression: $f(10) = -20 -$________. Explain your reasoning.

2. A sequence is defined by $f(0) = -4$, $f(n) = f(n-1) - 2$ for $n \geq 1$. Write a definition for the n^{th} term of the sequence.

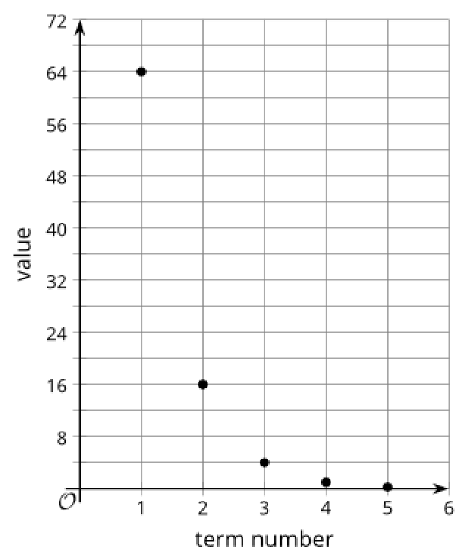
3. A sequence is defined by the explicit function $f(n) = 3n + 5$.

   a. Complete the table to show the first four terms of the sequence.

n	0	1	2	3
$f(n)$				

   b. Write the recursive equation f_n that defines the sequence. Explain your reasoning.

4. Here is the graph of a sequence:

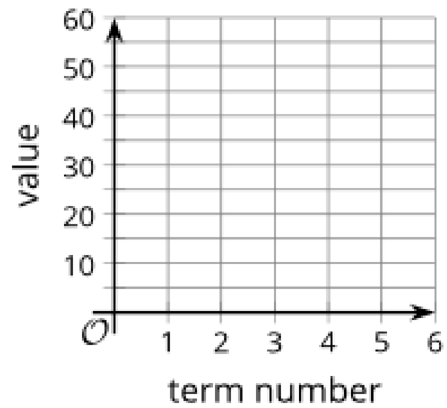


   a. Is this sequence arithmetic or geometric? Explain how you know.

   b. List at least the first five terms of the sequence.

   c. Write a recursive definition of the sequence.

5. Here is the recursive definition of a sequence: $f(1) = 3$, $f(n) = 2 \cdot f(n-1)$ for $n \geq 2$.



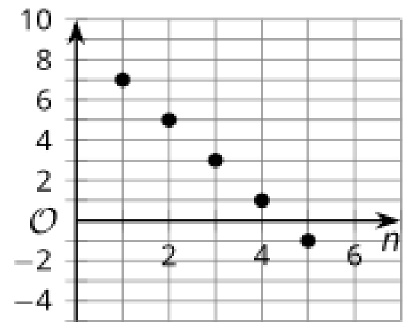
   a. Find the first five terms of the sequence.

   b. Graph the value of the term as a function of the term number.

   c. Is the sequence arithmetic, geometric, or neither? Explain how you know.

(From Unit 8, Lesson 5)

6. Here is a graph of sequence M. Define M recursively using function notation.



(From Unit 8, Lesson 5)

7. Write the first five terms of each sequence. Determine whether each sequence is arithmetic, geometric, or neither.

   a. $a(1) = 5$, $a(n) = a(n-1) + 3$ for $n \geq 2$.

   b. $b(1) = 1$, $b(n) = 3 \cdot b(n-1)$ for $n \geq 2$.

   c. $c(1) = 3$, $c(n) = -c(n-1) + 1$ for $n \geq 2$.

   d. $d(1) = 5$, $d(n) = d(n-1) + n$ for $n \geq 2$.

(From Unit 8, Lesson 4)

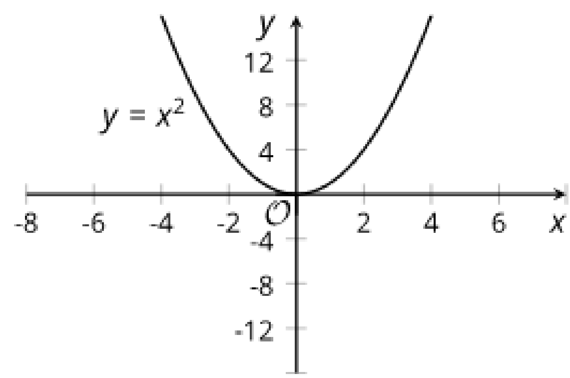
8. Four students solved the equation $x^2 + 225 = 0$. Their work is shown here. Only one student solved it correctly.

Student A:	Student B:	Student C:	Student D:
$x^2 + 225 = 0$ $x^2 = -225$ $x = 15$ or $x = -15$	$x^2 + 225 = 0$ $x^2 = -225$ No solutions	$x^2 + 225 = 0$ $(x - 15)(x + 15) = 0$ $x = 15$ or $x = -15$	$x^2 + 225 = 0$ $x^2 = 225$ $x = 15$ or $x = -15$

Determine which student solved the equation correctly. For each of the incorrect solutions, explain the mistake.

(From Unit 7)

9. Here is a graph that represents $y = x^2$.



   a. Describe what would happen to the graph if the original version was changed to:

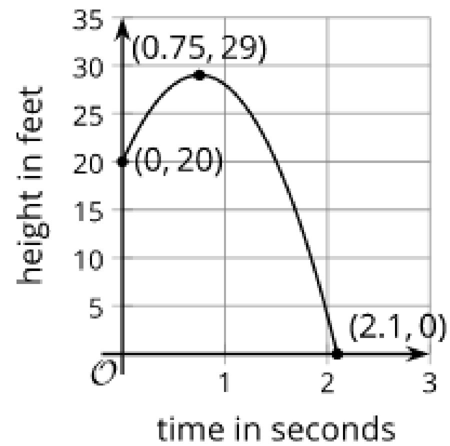
      i. $y = \frac{1}{2}x^2$

      ii. $y = x^2 - 8$

   b. Graph the equation $y = \frac{1}{2}x^2 - 8$ on the same coordinate plane as $y = x^2$.

(From Unit 7)

10. Clare throws a rock into the lake. The graph shows the rock's height above the water, in feet, as a function of time in seconds.



Select **all** the statements that describe this situation.

a. The vertex of the graph is $(0.75, 29)$.

b. The y-intercept of the graph is $(2.1, 0)$.

c. Clare dropped the rock into the lake without throwing it upwards.

d. The maximum height of the rock is about 20 feet.

e. The rock hits the surface of the water after about 2.1 seconds.

f. Clare tossed the rock up into their air from a point 20 feet above the water.

(From Unit 7)

11. (*Technology required*). Two objects are launched into the air. In both functions, t is seconds after launch.

- The height, in feet, of object A is given by the equation $f(t) = 4 + 32t - 16t^2$.
- The height, in feet, of object B is given by the equation $g(t) = 2.5 + 40t - 16t^2$.

Use technology to graph each function in the same graphing window.

a. What is the maximum height of each object?

b. Which object hits the ground first? Explain how you know.

(From Unit 7)

12. Jada and Lin were born on the same day to different families. Each family has grandparents that want to contribute money to an account for them to use when they complete high school.

Lin's grandparents have chosen to contribute $500 on the day he is born and then $100 per year until he is 18. Let $f(x)$ represent the amount of money in Lin's account after x years. Jada's grandparents have chosen to contribute $650 on the day he is born into an account that accrues interest, but they will never add any more money. The amount of money in Jada's account can be seen in the table to the right:

x	$g(x)$
0	650
2	716.625
4	790.07906
6	871.06217
8	960.34604
10	1058.7815
12	1167.3066
14	1286.9555
16	1418.8685
18	1564.3025

Compare and interpret the following features of each baby's account:

a. Vertical intercepts

b. $f(8)$ and $g(8)$

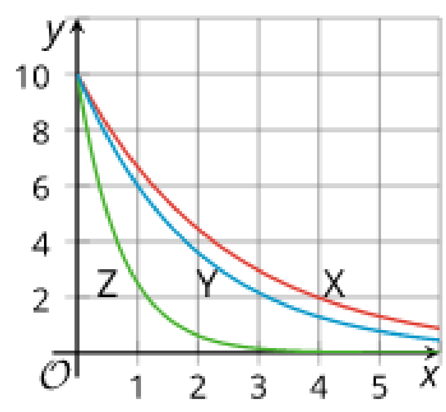
c. Average rate of change on the interval 0 to 4

d. Amount of money in the account when each person turns 18

(From Unit 7)

13. Here are the graphs of three equations.

Match each graph with the appropriate equation.



a. $y = 10(\frac{2}{3})^x$

b. $y = 10(\frac{1}{4})^x$

c. $y = 10(\frac{3}{5})^x$

1. Graph X
2. Graph Y
3. Graph Z

(From Unit 6)

Lesson 7: Situations and Sequences

Learning Targets

- I can identify the domain of a sequence.
- I can represent situations with sequences.

Warm-up: Describing Growth

1. Here is a geometric sequence: 16, 24, 36, 54, 81

   What is the growth factor?

2. One way to describe its growth is to say it's growing by ____% each time. What number goes in the blank for the sequence 16, 24, 36, 54, 81? Be prepared to explain your reasoning.

Activity 1: Finding Population Patterns

The table shows two animal populations growing over time.

Years since 1990	Population A	Population B
0	23,000	3,125
1	29,000	3,750
2	35,000	4,500
3	41,000	54,000

1. Are the sequences represented by population A and population B arithmetic or geometric? Explain how you know.

2. Write a definition for population A.

3. Write a definition for population B.

4. Does population B ever overtake population A? If so, when? Explain how you know.

Activity 2: Take the Cake!

A large cake is in a room. The first person who comes in takes $\frac{1}{3}$ of the cake. Then a second person takes $\frac{1}{3}$ of what is left. Then a third person takes $\frac{1}{3}$ of what is left. And so on.

1. Complete the table for $C(n)$, the fraction of the original cake left after n people take some.

n	$C(n)$
0	
1	$\frac{2}{3}$
2	
3	
4	

2. Write two definitions for C: one recursive and one non-recursive.

3. Construct a graph of the situation, giving special attention to the possible input values.

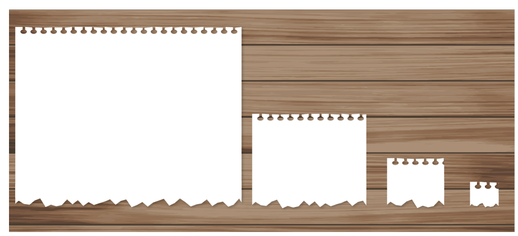
4. What is a reasonable domain for this function? Be prepared to explain your reasoning.

Lesson Debrief

Lesson 7 Summary and Glossary

The model we use for a function can depend on what we want to do.

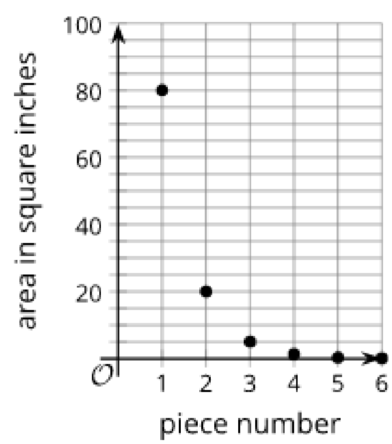
For example, an 8-inch-by-10-inch piece of paper has an area of 80 square inches. Picture a set of pieces of paper, each half the length and half the width of the previous piece.



Define the sequence A so that $A(n)$ is the area, in square inches, of the n^{th} piece. Each new area is $\frac{1}{4}$ the previous area, so we can define A recursively as:

$$A_1 = 80,\ A_n = A_{n-1} \cdot \frac{1}{4} \text{ for } n \geq 2$$

But for n-values larger than 5 or 6, the model isn't realistic since cutting a sheet of paper accurately when it is less than $\frac{1}{50}$ of a square inch isn't something we can do well with a pair of scissors. We can see this by looking at the graph of $y = A(n)$ shown here.



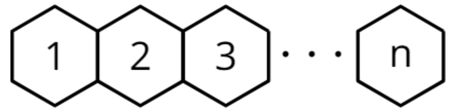
If we wanted to define the n^{th} term of A, it's helpful to first notice that the area of the n^{th} piece is given by $80 \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \ldots \cdot \frac{1}{4}$, where there are $n-1$ factors of $\frac{1}{4}$. Then we can write

$$A(n) = 80 \cdot \left(\frac{1}{4}\right)^{n-1},\ n \geq 1$$

We can use this definition to calculate a value of A without having to calculate all the ones that came before it. But since there are fewer than 10 values that make sense for A, since we can't cut very tiny pieces using scissors, in this situation we could just use the first definition we found to calculate different values of A.

Unit 8 Lesson 7 Practice Problems

1. A party will have hexagonal tables placed together with space for one person on each open side.



   a. Complete this table showing the number of people $P(n)$ who can sit at n tables.

n	1	2	3	4	5
$P(n)$	6				

   b. Describe how the number of people who can sit at the tables changes with each step.

   c. Explain why $P(3.2)$ does not make sense in this scenario.

   d. Define P recursively and for the n^{th} term.

2. Diego is making a stack of pennies. He starts with five pennies and then adds one penny at a time. A penny is 1.52 mm thick.

   a. Complete the table with the height of the stack $h(n)$, in mm, after n pennies have been added.

n	$h(n)$
0	7.6
1	
2	
3	

   b. Does $h(1.52)$ make sense? Explain how you know.

3. A piece of paper has an area of 80 square inches. A person cuts off $\frac{1}{4}$ of the piece of paper. Then a second person cuts off $\frac{1}{4}$ of the remaining paper. A third person cuts off $\frac{1}{4}$ what is left, and so on.

   a. Complete the table where $A(n)$ is the area, in square inches, of the remaining paper after the n^{th} person cuts off their fraction.

n	$A(n)$
0	80
1	
2	
3	

   b. Define A for the n^{th} term.

   c. What is a reasonable domain for the function A? Explain how you know.

4. A piece of paper is 0.05 mm thick.

   a. Complete the table with the thickness of the paper $t(n)$, in mm, after it has been folded n times.

n	$t(n)$
0	0.05
1	
2	
3	

   b. Does $t(0.5)$ make sense? Explain how you know.

5. A piece of paper has an area of 96 square inches.

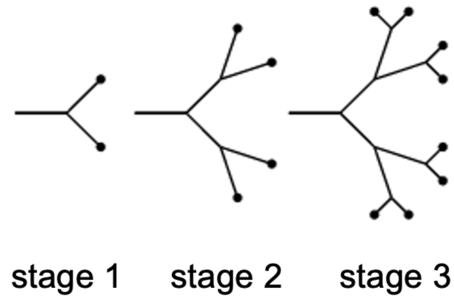
   a. Complete the table with the area of the piece of paper $A(n)$, in square inches, after it is folded in half n times.

n	$A(n)$
0	96
1	
2	
3	

   b. Define A for the n^{th} term.

   c. What is a reasonable domain for the function A? Explain how you know.

6. Here is a growing pattern:



   a. Describe how the number of dots increases from stage 1 to stage 3.

   b. Write a definition for sequence D, so that $D(n)$ is the number of dots in stage n.

   c. Is D a geometric sequence, an arithmetic sequence, or neither? Explain how you know.

7. A paper clip weighs 0.5 grams, and an empty envelope weighs 6.75 grams.

   a. Han adds paper clips one at a time to an empty envelope. Complete the table with the weight of the envelope, $w(n)$, in grams, after n paper clips have been added.

n	$w(n)$
0	6.75
1	
2	
3	

   b. Does $w(10.25)$ make sense? Explain how you know.

8.

a. An arithmetic sequence has $a(1) = 4$ and $a(2) = 16$. Explain or show how to find the value of $a(15)$.

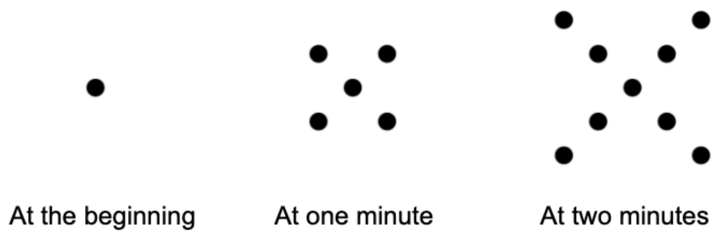
b. A geometric sequence has $g(0) = 4$ and $g(1) = 16$. Explain or show how to find the value of $g(15)$.

(From Unit 8, Lesson 6)

9. An arithmetic sequence k starts 4, 13, ….Explain how you would calculate the value of the 5,000th term.

(From Unit 8, Lesson 6)

10.



a. Describe the pattern that you see in the sequence of the figures above.[1]

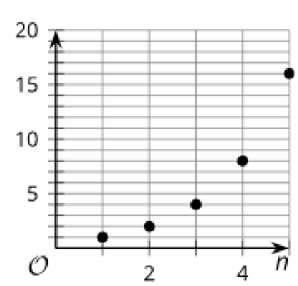
b. Assuming the pattern continues in the same way, how many dots are there at 3 minutes?

c. How many dots are there at 100 minutes?

d. How many dots are there at t minutes? Solve the problem by your preferred method. Your solution should indicate how many dots will be in the pattern at 3 minutes, 100 minutes, and t minutes. Be sure to show how your solution relates to the picture and how you arrived at your solution.

(From Unit 8, Lesson 6)

11. Here is a graph of sequence t. Define t recursively using function notation.



(From Unit 8, Lesson 5)

[1] Adapted from Open Up Resources. Access the full curriculum, supporting tools, and educator communities at openupresources.org.

12. Match each sequence with one of the recursive definitions. Note that only the part of the definition showing the relationship between the current term and the previous term is given so as not to give away the solutions. One of the sequences matches two recursive definitions.

a. $a(n) = a(n-1) - 4$

b. $b(n) = b(n-1) + 0$

c. $c(n) = -\frac{1}{2} \cdot c(n-1)$

d. $d(n) = 1 \cdot d(n-1)$

1. 7, 3, -1, -5
2. $1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}$
3. 8, 8, 8, 8

(From Unit 8, Lesson 4)

Lessons 8 & 9: Mathematical Modeling[1]

Learning Targets

- I can use mathematics to model real-world situations.
- I can test and improve mathematical models for accuracy in representing and predicting real things.

Advice on Modeling

These are some steps that successful modelers often take and questions that they ask themselves. You don't necessarily have to do all of these steps, or do them in order. Only do the parts that you think will help you make progress.

	Understand the Question Think about what the question means before you start making a strategy to answer it. Are there words you want to look up? Does the scenario make sense? Is there anything you want to get clearer on before you start? Ask your classmates or teacher if you need to.
	Refine the Question If necessary, rewrite the question you are trying to answer so that it is more specific.
	Estimate a Reasonable Answer If you don't have enough information to decide what's reasonable, try to come up with an answer that would be too low, and an answer that would be too high.
	Identify Unknowns ● What are the meaningful quantities in this situation? Write them down. ● What information would be useful to know? In order to get that information, you could: look it up, take a measurement, or make an assumption.
	Gather Information Write down any of the unknown information that you find. Organize your information in a way that makes sense to you.
	Experiment! Try different ideas to make progress toward answering your question. If you are stuck, think about: ● Helpful ways to organize the information you have or organize your work ● Questions you *can* answer using the information you have ● Ways to represent mathematical relationships or sets of data (tables, equations, graphs, statistical plots) ● Tools that are available for representing mathematics, both digital and analog
	Check Your Reasoning Do you have a first answer to your question? Great! See if it's reasonable. ● Make sure you can explain what the answer means in terms of the original problem. ● Check your precision: Is your answer overly precise? Not precise enough?
	Use and Improve Your Model ● Did you make assumptions or measurements? How can you express your model more generally, so that it would work for a range of numbers instead of the specific numbers you used? ● What are the limitations of your model? That is, what are some ways it is not realistic? Does it only work for certain inputs but not others? Are there any meaningful inputs affecting the outcome that are not accounted for? If possible, improve your model to take these into account. ● What are the implications of your model? That is, what should people or organizations do differently or smarter as a result of what your model shows? What would be effective ways to communicate with them? ● What are the areas for further research? That is, what new things are you wondering about that could be investigated, by you or someone else?

[1] Adapted from IM 9–12 Math Algebra 1 Modeling Prompts https://curriculum.illustrativemathematics.org/HS/index.html,

Modeling Rubric

<table>
<tr><th rowspan="2">Skill</th><th colspan="3">Score</th><th rowspan="2">Notes or Comments</th></tr>
<tr><th>Proficient</th><th>Developing</th><th>Needs Revisiting</th></tr>
<tr><td rowspan="2">1. Decide What to Model</td><td>● Assumptions made are clearly identified and justified. Resulting limitations are stated when appropriate.
● Variables of interest are clearly identified and chosen wisely, and appropriate units of measure are used.</td><td>● Assumptions are noted but lacking in justification or difficult to find.
● Variables of interest are noted, but may lack justification, be difficult to find, or not be measured with appropriate units.</td><td>● No assumptions are stated.
● No variables are defined.</td><td></td></tr>
<tr><td colspan="4">To improve at this skill, you could:
● Ask questions about the situation to understand it better
● Check the assumptions you're making to see if they're reasonable (Try asking a friend, or imagining that you're a person involved in the scenario. Would those assumptions make sense to you?)
● Double-check the variables you've identified: Are there other quantities in the situation that could vary? Is there something you've identified as a variable that is actually fixed or determined? (Remember that more abstract things like time and speed are also quantities.)</td></tr>
<tr><td rowspan="2">2. Formulate a Mathematical Model</td><td>● An appropriate model is chosen and represented clearly.
● Diagrams, graphs, etc. are clear and appropriately labeled.</td><td>Parts of the model are unclear, incomplete, or contain mistakes.</td><td>No model is presented, or the presentation contains significant errors.</td><td></td></tr>
<tr><td colspan="4">To improve at this skill, you could:
● Check your model more carefully to make sure it really fits well
● Consider a wider variety of possible models, to find one that fits the situation better
● Think about the situation more deeply before trying to find a model
● Convince a skeptic: Pretend that you think your model is inadequate, or ask a friend to pretend to be skeptical of it. What would a skeptic find wrong with your model? Try to fix those things, or explain why they're not actually problems.</td></tr>
</table>

<table>
<tr><th rowspan="2">Skill</th><th colspan="3">Score</th><th rowspan="2">Notes or Comments</th></tr>
<tr><th>Proficient</th><th>Developing</th><th>Needs Revisiting</th></tr>
<tr><td rowspan="2">3. Use Your Model to Reach a Conclusion</td><td>• Solution is relevant to the original problem.
• Reader can easily understand the reasoning leading to the solution.
• Relevant details are included like units of measure.</td><td>Solution is not well-aligned to the original problem, or aspects of the solution are difficult to understand or incomplete.</td><td>No solution is provided.</td><td></td></tr>
<tr><td colspan="4">To improve at this skill, you could:
• Double-check your calculations: Show them to someone else to see if they agree, or take a break and look at your calculations again later
• Make sure your calculations are justified by your model: Ask yourself how you decided what to calculate, and see if your reasoning matches up with your model
• Think more deeply about what your conclusions mean in the original scenario: Imagine you're a person involved in the scenario, or explain your conclusions to someone else and see if they have questions</td></tr>
<tr><td rowspan="2">4. Refine and Share Your Model</td><td>• The model’s implications are clearly stated.
• The limitations of the model and solution are addressed.</td><td>The limitations of the model and solution are addressed but lacking in depth or ignoring key components.</td><td>No interpretation of model and solution is provided.</td><td></td></tr>
<tr><td colspan="4">To improve at this skill, you could:
• Think more creatively about what your conclusions mean: Ask yourself "If I was involved in this situation, what would I understand better because of these conclusions? What would I want to do next?"
• Be skeptical of your model: What don't you like about it, and what can you do to fix those things?
• Explain your model to someone else: Tell them how it works and why it's good. If you're not sure how it works or why it's good, you might need to change it.</td></tr>
</table>

Workspace for Modeling Prompt # _____

Workspace for Modeling Prompt # _____

Modeling Prompt # ____ Reflection